2nd GAPCOMB Workshop

Montserrat, July 12-14, 2021

Programme

Monday, July 12th

16:00-16:30	Welcome and presentation of the workshop
16:30-17:00	Talk 1: Patrick Morris
17:00-17:30	Coffee Break
17:30-18:15	3 talks by master students
18:15-18:30	Break
18:30-19:00	Talk 2: Clément Requilé
19:00-20:15	Free time
20:15-21:15	Dinner
21:30-23:00	Open problem session. Chair: Lluís Vena

Tuesday, July 13th

8:30-9:30	Breakfast
9:30-10:00	Talk 3: Maximilian Wötzel
10:00-10:30	Short Break
10:30-11:30	Work in open problems
11:30-12:00	Coffee break
12:00-13:30	Work in open problems
13:30-16:00	Lunch and Free time
16:00-17:00	Business meeting
17:00-17:30	Coffee break
17:30-18:00	Talk 4: Ricard Vilar
18:00-19:30	Work in open problems
19:30-20:30	Visit to Escolania
20:30-21:20	Dinner

Wednesday, July 14th

8:30-9:30	Breakfast
9:30-10:00	Talk 5: Matthew Coulson
10:00-10:15	Short Break
10:15-11:30	Work in open problems
11:30-12:00	Coffee break
12:00-13:30	Conclusions on the open problems
13:30-15:30	Lunch and departure

Monday at 16:30:

Triangle factors in pseudorandom graphs

Patrick Morris

An (n, d, λ)-graph is an n vertex, d-regular graph with second eigenvalue in absolute value λ. When λ is small compared to d, such graphs have *pseudorandom* properties. A fundamental question in the study of pseudorandom graphs is to find conditions on the parameters that guarantee the existence of a certain subgraph. A celebrated construction due to Alon gives a triangle-free (n, d, λ)-graph with $d = \Theta(n^{2/3})$ and $\lambda = \Theta(d^2/n)$. This construction is optimal as having $\lambda = o(d^2/n)$ guarantees the existence of a triangle in an (n, d, λ)-graph. Krivelevich, Sudakov and Szabó (2004) conjectured that if $n \in 3\mathbb{N}$ and $\lambda = o(d^2/n)$ then an (n, d, λ)-graph G in fact contains a *triangle factor*: vertex disjoint triangles covering the whole vertex set.

In this talk, we discuss a solution to the conjecture of Krivelevich, Sudakov and Szabó. The result can be seen as a clear distinction between pseudorandom graphs and random graphs, showing that essentially the same pseudorandom condition that ensures a triangle in a graph actually guarantees a triangle factor. In fact, even more is true: as a corollary to this result and a result of Han, Kohayakawa, Person and the author, we can conclude that the same condition actually guarantees that such a graph G contains every graph on n vertices with maximum degree at most 2.

Monday at 17:30:

The Gap between an Automorphism and its Inverse

Àlex Miranda

We reintroduce the function α_G (and β_G) that measures the gap between an (outer) automorphism of G and its inverse. We give an alternative proof of the lower bound for α_{F_r} of the free groups, and give an improvement for the lower bound of β_{F_r}. Furthermore, for the first time, a study of the function $\alpha_{\mathrm{BS}(1,N)}$ for the Baumslag-Solitar groups $\mathrm{BS}(1, N)$, $|N| > 1$, is made, and we prove that it grows linearly. Finally, in an independent way, we define the same concept over the virtual automorphisms and prove that the equivalent function for the free groups has an exponential lower bound.

Monday at 17:45:

Extremal Sidon Sets are Fourier Uniform

Miquel Ortega

We prove that extremal Sidon sets are Fourier uniform. From this we recover in a unified manner previously known results on their equidistribution over arithmetic progressions. As a further application we deduce that, for any partition regular equation in five or more variables, every finite colouring of an extremal Sidon set has a monochromatic solution.

Monday at 18:00:

Bijective enumeration of constellations in higher genus

Jordi Castellví

Bousquet-Mélou and Schaeffer gave in 2000 a bijective enumeration of some planar maps called constellations. In 2019, Lepoutre described a bijection between bicolorable maps of arbitrary genus and some unicellular maps of the same genus.

We present a bijection between constellations of higher genus and some unicellular maps that generalizes both existing bijections at the same time.

Using this bijection, we manage to enumerate a subclass of constellations on the torus, proving that its generating function is a rational function of the generating function of some trees.

Monday at 18:30:

From the Ising model on maps to the enumeration of bipartite planar graphs

Clément Requilé

The asymptotic enumeration of labelled planar graphs has a rather long history. It started with Tutte in the 60's, with the enumeration of planar maps (fixed embedding of connected planar multigraphs), and was concluded by Gimnez and Noy in 2009. In the meantime, several authors studied families of planar maps or graphs with a global constraint, such as the degree (regular, eulerian, etc.), the connectivity, the hamiltonicity, etc.. For instance, the enumeration of bipartite planar maps comes directly from that of eulerian maps and is now a classical exercise. However, the enumeration of bipartite planar graphs seems to be more involved.

The Ising model on a map represents all possible 2-colourings of its vertices. Those colourings are not necessarily proper in the sense that we allow monochromatic edges.

In combinatorial terms we consider the Ising generating function associated to a class of maps, whose coefficients count every 2-coloured maps with a weigth u^m, where m is the number of monochromatic edges. Bernadi and Bousquet-Mlou proved in 2011 that this generating function is always algebraic. In this talk we will discuss how this leads to the asymptotic enumeration of labelled bipartite planar graphs.

This is (ongoing) joint work with Marc Noy and Juanjo Rué.

`Tuesday at 9:30:`

An approximate structure theorem for sets with small sumset

Maximilian Wötzel

Freiman's theorem tells us that any integer set A with bounded doubling constant K is contained in a multidimensional arithmetic progression of size and dimension only depending on K, but can something stronger be said about almost all such sets?

A recent result of Campos shows that yes, with high probability, a random set A of integers in the interval $[0, N]$ chosen among subsets with given cardinality s and doubling constant K is almost contained in a short arithmetic progression of size $Ks/2$. Surprisingly, this result holds even outside the case of bounded doubling, as K is allowed to be of the order $s/\log(n)^3$.

We extend this result to pairs of distinct sets, obtaining equivalent bounds when the sets' sizes are within a constant factor of each other. The main tool used in the proof is an extension of the asymmetric container lemma recently introduced by Morris, Samotij and Saxton to multipartite hypergraphs.

Joint work with Marcelo Campos, Matthew Coulson and Oriol Serra.

`Tueday at 17:30:`

Determining when a truncated generalised Reed-Solomon code is Hermitian self-orthogonal

Ricard Vilar

We prove that there is a Hermitian self-orthogonal k-dimensional truncated generalised Reed-Solomon of length $n \leqslant q^2$ over $\mathbb{F}_{q^2}$ if and only if there is a polynomial $g \in \mathbb{F}_{q^2}$ of degree at most $(q-k)q-1$ such that $g+g^q$ has q^2-n zeros. This allows us to determine the smallest n for which there is a Hermitian self-orthogonal k-dimensional truncated generalised Reed-Solomon of length n over $\mathbb{F}_{q^2}$, verifying a conjecture of Grassl and Rötteler. We also provide examples of Hermitian self-orthogonal k-dimensional generalised Reed-Solomon codes of length q^2+1 over $\mathbb{F}_{q^2}$, for $k = q-1$ and q an odd power of two. And the motivation for studying Hermitian self-orthogonal codes is that they give Quantum codes.

Wednesday at 9:30:

Components of barely subcritical random digraphs

Matthew Coulson

One of the foundational questions in the theory of random graphs concerns the size of the largest component of such a graph. For random digraphs an analogous question asks what is the size of the largest strong component? We consider this question in the directed configuration model which is a model for random digraphs with a fixed degree sequence: each vertex v has a specified in-degree d_v^- and out-degree d_v^+. Define

$$Q := \frac{\sum_v d_v^- d_v^+}{\sum_v d_v^+} - 1$$

We show that in the barely subcritical regime, the largest strong component is a directed cycle of size $\Theta(1/|Q|)$ and furthermore, that the probability the kth largest strong component has size at most $\alpha/|Q|$ is asymptotically equal to

$$\sum_{i=0}^{k-1} \frac{\xi_\alpha^i}{i!} e^{-\xi_\alpha} \quad \text{where} \quad \xi_\alpha := \int_\alpha^\infty \frac{e^{-\lambda}}{\lambda} d\lambda.$$

This generalises a result of Łuczak and Seierstad who showed a similar phenomenon in the model $D(n,p)$.

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