Odd Cycle Games and Connected Rules

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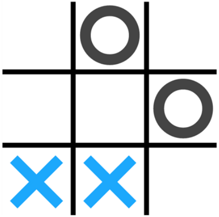

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For what values of b do Breaker and Waiter win?

The point where the winner switches is referred to as the **bias threshold**, denoted by b_{mb} and b_{cw}.

Examples of Positional Games

Let the board X be given by all edges of the complete graph on n vertices.

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The winning sets of the **connectivity game** consist of all spanning trees of K_n. Gebauer and Szabó showed that $b_{\mathrm{mb}} \approx n/\ln n$.

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The winning sets of the **cycle game** are all cycles in K_n. In the **odd (even) cycle game** the winning sets are all odd (even) cycles.

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Theorem 1 (Corsten, Mond, Pokrovskiy, S. and Szabó ′19+)

In the Maker-Breaker odd cycle game

$$b_{mb} \geq \left(\frac{4-\sqrt{6}}{5} - o(1) \right) n \approx 0.3101n.$$

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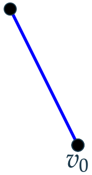

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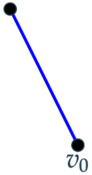

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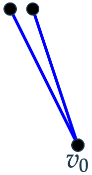

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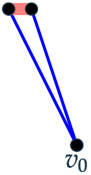

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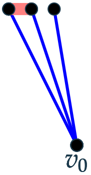

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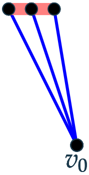

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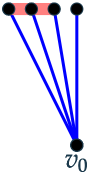

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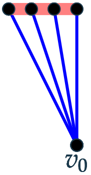

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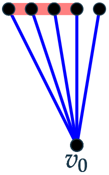

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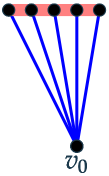

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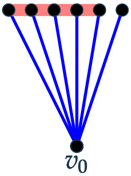

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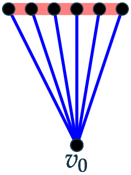

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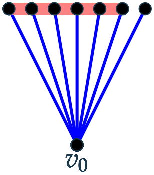

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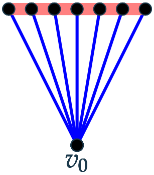

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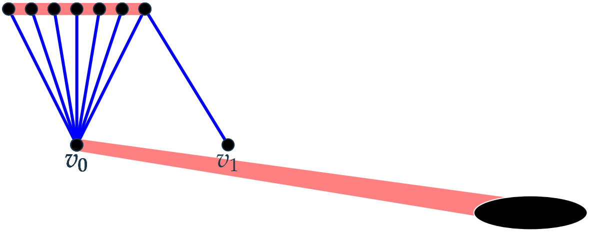

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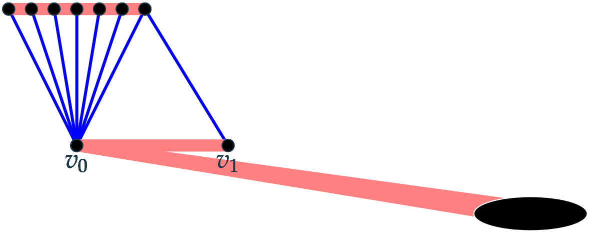

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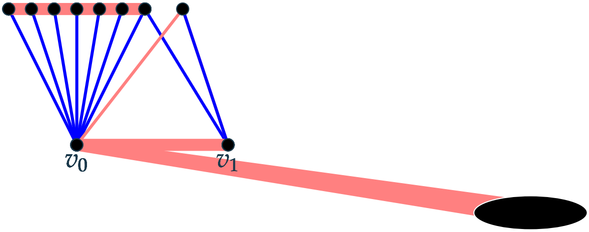

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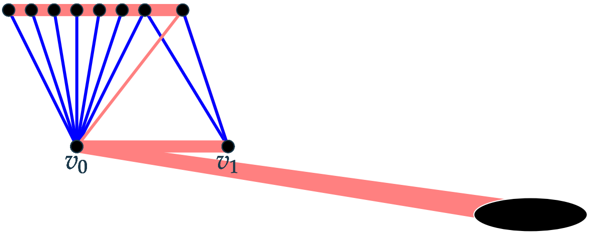

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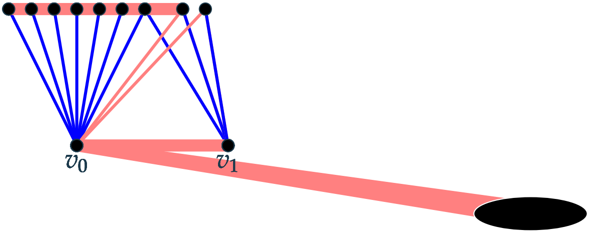

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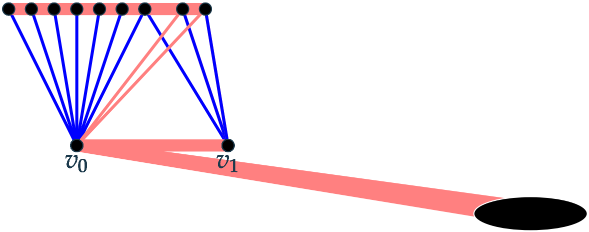

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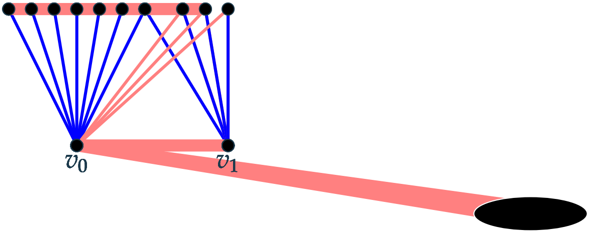

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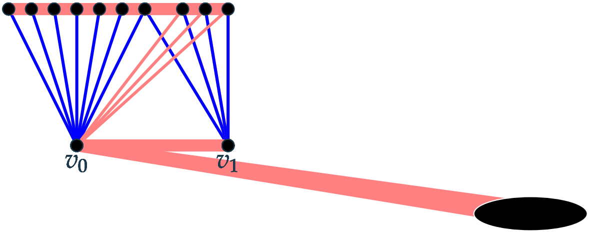

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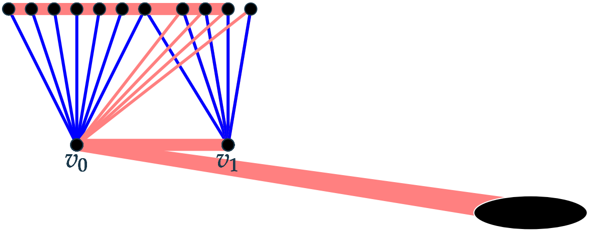

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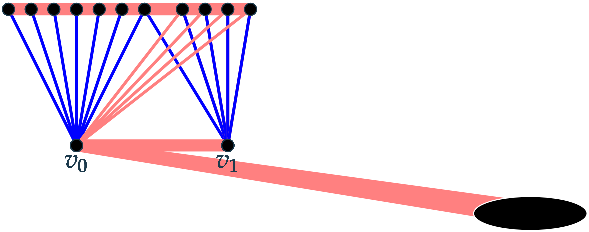

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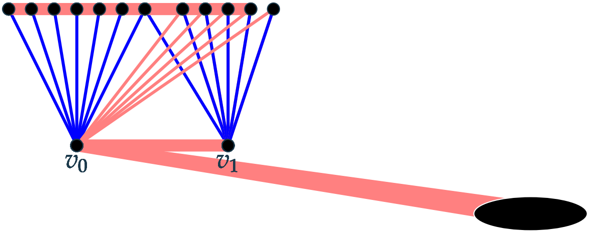

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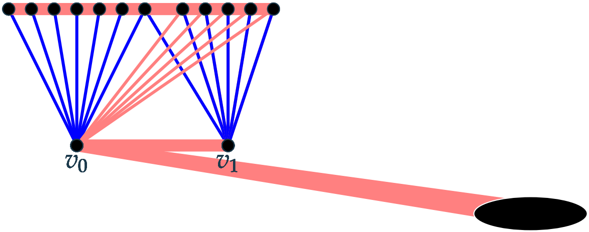

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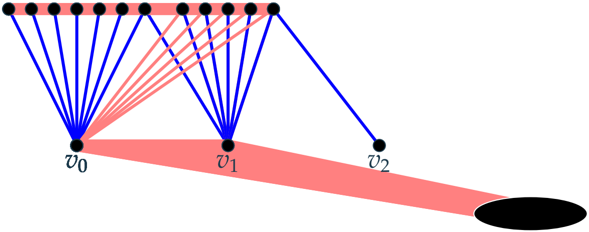

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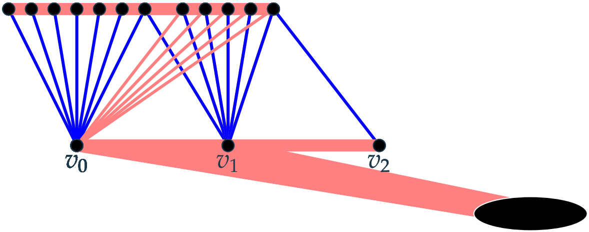

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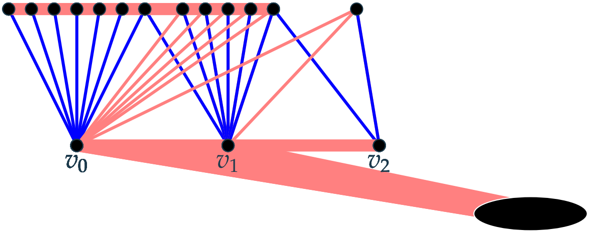

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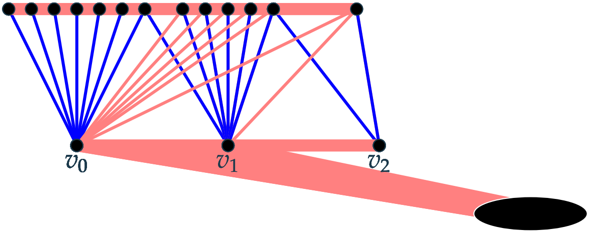

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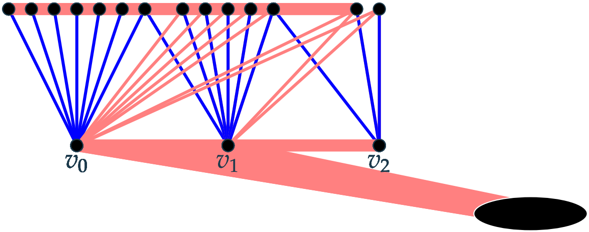

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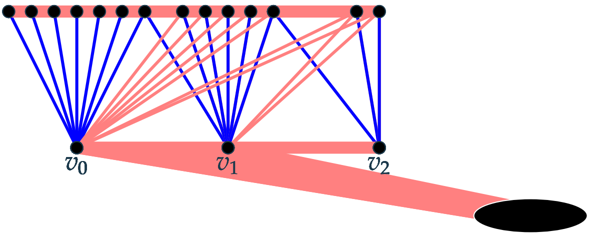

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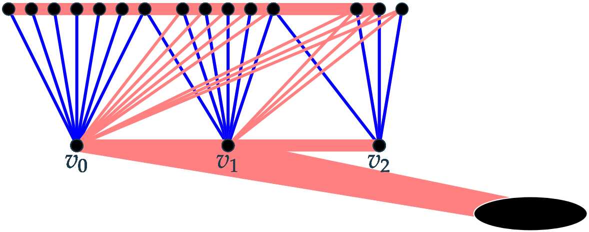

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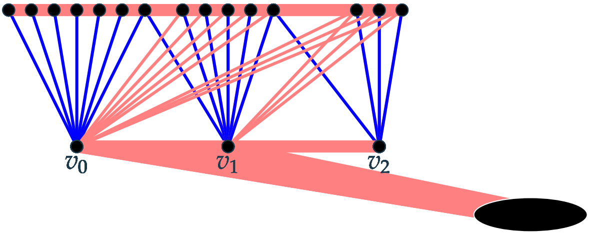

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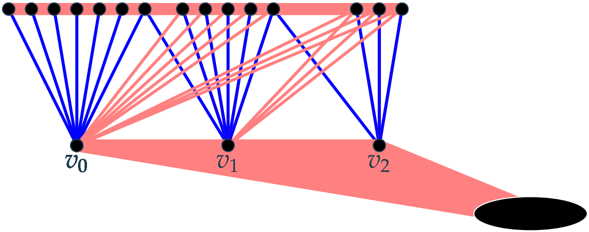

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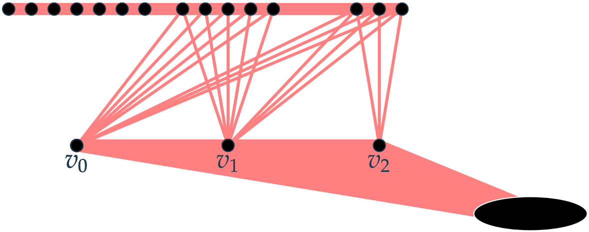

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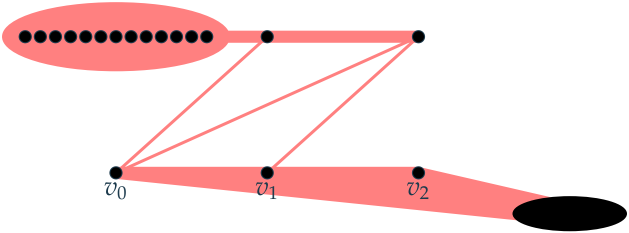

Connected Maker-Breaker Cycle Games

Theorem (Bednarska and Pikhurko 2008)

In the Maker-Breaker odd cycle game $b_{mb} \geq 0.2928n$.

Theorem 1 (Corsten, Mond, Pokrovskiy, S. and Szabó '19+)

In the Maker-Breaker odd cycle game $b_{mb} \geq 0.3101n$.

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A Strategy for Breaker under Connected Rules

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3. This way Breaker minimises the number of edges ending up between the two parts of Maker's graph.

Client-Waiter Cycle Games

Theorem (Hefetz, Krivelevich, and Tan 2016)

In the Client-Waiter cycle game $b_{cw} = \lceil n/2 \rceil - 1$.

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In the Client-Waiter **odd** *cycle game* $b_{cw} \geq n/(4 \log 2) - o(n) \approx 0.3606n$.

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Conjecture (Hefetz, Krivelevich and Tan)

We have $b_{cw} = n/2 - o(n)$ *in the odd cycle Client-Waiter game.*

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Proof Idea

1. Client's graph will again be bipartite as long as she hasn't won the game.
2. If at any point there is an unclaimed edge inside either of the two parts, Waiter will loose.
3. Whenever offering an edge incident to a vertex not yet in Client's graph, Waiter must either *offer all unclaimed edges between that vertex and Client's graph*

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4. Client tries to reduce the number of times the later occurs.

Open Question

Q1. What is the threshold bias for other variants of the odd cycle games, for example Avoider-Enforcer or Waiter-Client?

Q2. What is the threshold bias for the connected Maker-Breaker H-game?

Q3. One can view the odd cycle game as the non-2-colourability game. It was proved by Hefetz et al. that the threshold bias for the Maker-Breaker non-k-colourability game satisfies $b_{mb} = \Theta_k(n)$. Do we have $b_{mb} \approx b^c_{mb}$?

Thank you for your attention!